Breaking even

***When a business is first set up the owner must outlay an amount of money for which there is no immediate return. This is the money that pays for the initial purchase of machinery, materials and the like. This initial outlay is like a flag fall. Once the business is producing and selling goods, it has other costs as well as the initial costs, but also has an income. Hence the initial outlay can begin to be recouped. Most people are interested to know how much product they need to sell before they* break even – *that is when* income is equal to the initial outlay.**

Consider the following simple business situation.

> *Last holidays Tyron was bored. He decided to start a little business in which he made and sold skateboards.*
>
> *He had to set up his Dad's shed with tools and a bench which cost him $500. Each board he produced cost him $30 to make. Once made, he sold them for $50.*
>
> *He wondered how many boards he would have to sell before he broke even, that is until the income from selling boards equalled the initial outlay* **and** *the cost of making that number of boards.*

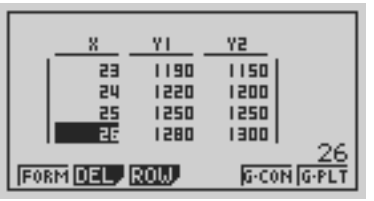
The cost (C) associated with the number of boards (b) that Tyron makes can be modelled by

$$C = 30b + 500.$$

The income (I) associated with the venture can be modelled by

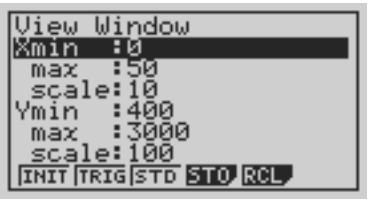
$$I = 50b.$$

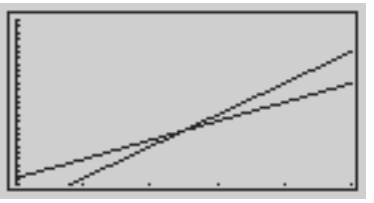
Define both of these functions in the TABLE mode of the calculator. Then produce a table that will allow us to see how many boards Tyron will need to sell to break even. Our table looks like the following.



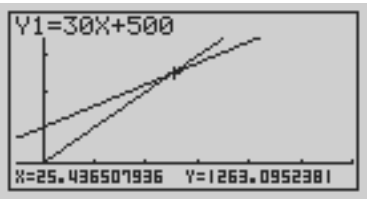
It looks like he must sell 25 skateboards to break even (ie. Y1 = Y2). We could discover this in another way.

Enter the GRAPH mode of your calculator. The two functions just defined will be present. Set the view window of the calculator as shown below. Now draw a graph of the two functions.

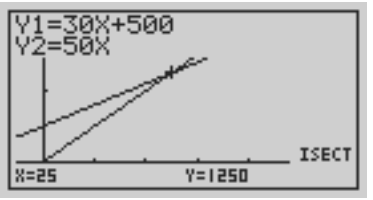




Note that the same values for cost and income that we saw in our table are illustrated on this graph as a cross over point (more formally called the *point of intersection)* of the two lines that illustrate the functions.



We could trace the lines (SHIFT then F1) to find the point of intersection. The accuracy of this method depends on the scale you are using. A more accurate way is to use the calculator function that automatically finds the coordinates of the point of intersection.



With the graph showing on your calculator, use G-Solv (SHIFT then F5) to access ISCT (F5). ISCT stands for intersection. Wait, and a moving cursor travels along one of the lines until it reaches the point of intersection.

Again we see that Tyron would need to make and sell 25 skateboards to break even. Both cost and income are $1250.

We could also achieve this result by solving an equation. Clearly to break even, income must equal cost, so

$$\begin{aligned}
& \text{income} &=& \text{cost} \\
\Rightarrow\quad & 50b &=& 30b + 500 \\
\Rightarrow\quad & 50b - 30b &=& 30b + 500 - 30b \\
\Rightarrow\quad & 20b &=& 500 \\
\Rightarrow\quad & \frac{20b}{20} &=& \frac{500}{20} \\
\Rightarrow\quad & b &=& 25
\end{aligned}$$

You now have three ways to solve a break even problem.

Interaction M

1. Determine how many boards Tyron will have to sell to break even if he sells the boards for:
   i) $35
   ii) $40
   iii) $60
2. What would the selling price need to be to break even after selling just one board?
3. What selling prices will result in Tyron never breaking even? Illustrate these situations graphically.
4. Tyron's dad decides his son is onto a good thing and decides to get into the business of making and selling skateboards. He sets up a factory at the cost of $40 000. He is able to make the boards at the cost of $20 per board. The selling price is set at $60 per board – they are a bit flasher than the Tyron originals. Determine how many boards that Tyron's dad must sell before he breaks even.