Data logging and cooling curves

Level
Upper secondary

Mathematical Ideas
Data logging, exponential functions, transformation of functions, mathematical modelling

Description and Rationale
Along with the advent of the graphics calculator and data collecting devices has come the ability to quickly, easily and cheaply design experiments that provide real data on which students can complete real life mathematical modelling tasks.

Much of the mathematics presently done in schools is removed from the real world and as such seen as not very relevant, with the data logger the student moves closer to the real world applications of mathematics. Students are motivated by the experience of collecting the information, analysing the data and reporting their findings. The opportunities for collaborative learning to occur are immense, with benefits for both the students and the teacher.

Hot water, placed in a cup, cools over time to room temperature. The rate of cooling depends on a number of factors, including the material the cup is made from. The cooling process itself is an interesting one to try and describe mathematically as a number of different types of functions can be developed that model cooling behaviour. This activity will explore the cooling behaviour of a probe in air alone so that the rate of cooling can be observed with a minimum of other factors that may effect the outcome. The cooling behaviour of other materials, under different conditions, may be investigated in follow up activities.

There are many methods and programs for collecting the data in an experiment such as this. GETDATA2 is a program written to control probes on the data logger from the calculator, it is available from the Casio website and is commonly used to run data loggers.

Equipment
You will need the following for this experiment:

coffee cup and hot water
graphics calculator
data logger and temperature probe
thermometer

The Task
Each group is asked to perform the experiment set out below to collect the required data. Each group is responsible for collecting their own data. Room temperature is recorded and then the variation of temperature with time is recorded after the probe is removed from the hot water.

Students may be asked to determine a model for the cooling, interpret the constants of the model and present their findings to the rest of the class

The Experiment

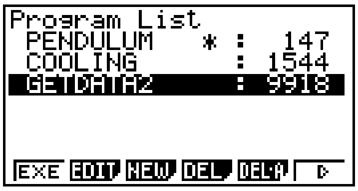
1. Connect your calculator to the data logger via the communication port and the temperature probe to Channel 1 on the data logger.

2. Record the room temperature

3. Fill your cup with boiling water

4. Place the Temperature probe in the water for approximately 2 minutes.

5. Go to the Program menu and choose GETDATA2.



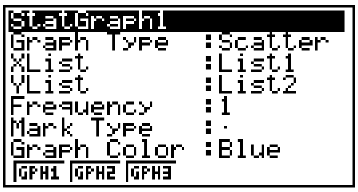
6. We need to set the probe type and the channel it is connected to (SetP) along with the number of samples and the time interval between samples (SetS). These need to be set according to the task being completed. Here we used a temperature probe (Temp) in CH1 and take 90 samples at 1 second intervals.

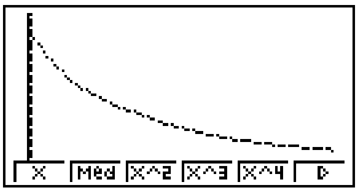
7. Remove the probe and press GO (F1)

8. When the data is collected transfer the data to the calculator, following the instructions on the calculator, observe the scatter plot of the data and attempt to model the data.

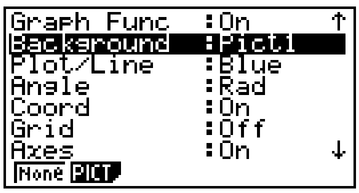
Finding the model

Produce a scatter plot of the data.





So that the data may be modelled a picture is placed in the background. This is done by choosing OPTN and then following the prompts to store a picture. The picture is then placed in the background via the SET-UP (SHIFT then MENU).



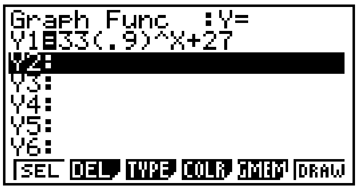
Students then use their knowledge of exponential functions and the transformation of functions to develop and refine a model that they believe fits the data to a satisfactory level. The room temperature was found to be approximately 27°for this data set.

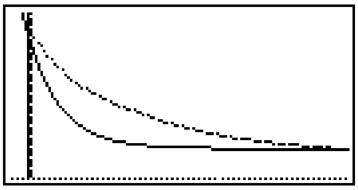
The basic form of one function that may fit the data is $T = a(b)^t + c$ where:

> *a* is the distance between the highest value and the lowest value (room temperature).
> *b* is the decay rate (a value between 0 and 1).
> *c* is the room temperature (the vertical transformation).

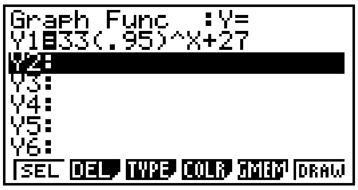
Based on the data collected a value for *a* in the initial model would be (60 – 27) = 33.
A value for *c* would be 27
b could be determined using the common ratio between successive terms or a value could be chosen and then adjusted as necessary, here we will try 0.90.

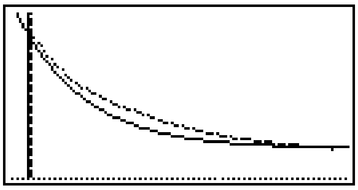
The function can be placed in the calculator and graphed.



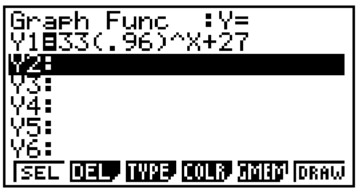


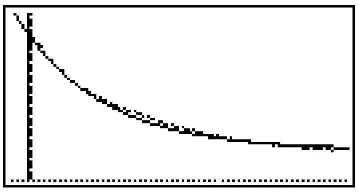
The function is appropriate at the beginning and end of the data but has dropped off too quickly in between. Students would be expected to realise that the decay rate is causing this difficulty and so a value closer to 1 needs to be used. Students explaining these ideas should be an integral component of the report to the class.





With a slight alteration in the decay rate a function that satisfactorily fits the data set can be achieved.





The more conventional expression of the exponential model is

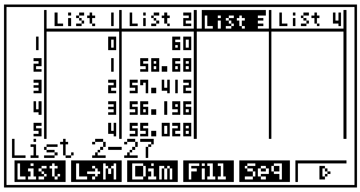
$$y = Ae^{kx} + c$$

where $b = e^k$, $(k = \ln b)$

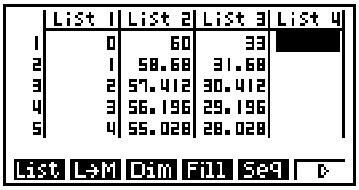
Using this model a result similar to that above could be generated.

Students may have also considered transforming the original data so that a simpler model may be fitted. With the data in the lists a new list, with the temperature lowered by the room temperature can be created.

To complete an operation on a list of numbers the list name must be highlighted. The available commands can be found via the OPTN button, then LIST menu. Here the room temperature is being subtracted from each recorded temperature.



The data in List 3 could now be modelled against List 1 to produce a model without the vertical transformation.



An activity such as this is an ideal opportunity for students to work collaboratively and to communicate how they developed their model, including the reasoning for the adjustments made along the way. Students should be encouraged to communicate, both orally and in written form, the mathematics involved in completing this activity. Some guidelines as to what is expected or required in the students' presentation is desirable for all parties concerned.

While only an exponential model has been investigated here the consideration of other types of models and the reasons for accepting or rejecting them could be an integral component of this activity. The exploration of the limitations of any models investigated is a valid component of any mathematical modelling activity.