Crossing the River

Level
Upper primary and lower secondary

Mathematical ideas
Problem solving, patterning, multiple representation, the need for restricting the domain in certain contexts

Description and rationale
This lesson is derived from the classic puzzle by Henry Ernest Dudeney. The puzzle reads:

> *During the Turkish stampede in Thrace, a small detachment found itself confronted by a wide and deep river that could only be crossed safely in a boat. However, they discovered a boat in which two children were rowing about. It was so small that it would carry only two children or one adult. The detachment consisted of 358 adults. How did the 358 adults cross the river and leave the two children in joint possession of the boat? How many trips (from bank to bank) were made if the least number of trips occurred?*

This puzzle has been changed a little to present to students. The task for the lesson is:

> *A number of adults and children stand on one side of a river that can only be crossed in a boat. They have a small boat that can hold one adult or two children. It is know that 87 trips (bank to bank) are made in order to get the group across the river. It is also know that the least number of trips possible is 87. How many adults and children are in the group?*

This problem offers a wonderful challenge to the students and will develop their abilities in problem solving and patterning. The graphics calculator plays only a minor role – as a medium to display the thinking of the student. However, when using the calculator the student is forced to consider things that traditionally they may not have.

After some thinking the students will be able to find *some* answers to this problem, probably by trial and error. It is likely students will get different answers, leading to the questions

- how many answers are there?
- is there a pattern to the answers?
- is there a simply way to write down all answers?

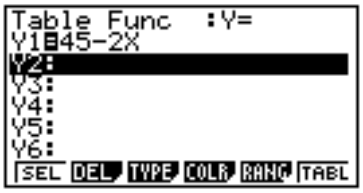
Student's that approach this problem with a mathematical strategy may have the answer to the above questions as a result of their work, trial and error technicians may not.

With some more effort students should be able to conjecture that if c is defined as the number of children and a as the number of adults, then

$$c = 45 - 2a$$

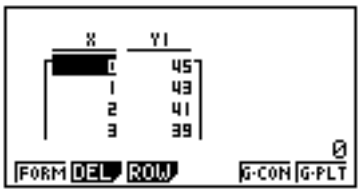
It is left to the reader to verify the truth of this conjecture.
At this point many may not have considered whether this rule is sensible for all values of a.

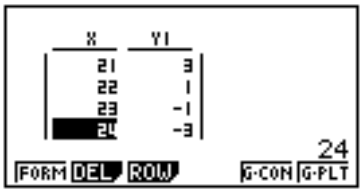
Once the students have seen the pattern and conjectured its rule they can use the graphics calculator to display the results in a number of ways.
Firstly they could produce a table of values by defining Y1 as the rule they generated.



Left to their own devices, the students may well not set the range (domain actually!!) for this rule. If they do, many may do it without thought, as seen opposite. They should be left to explore at this stage.

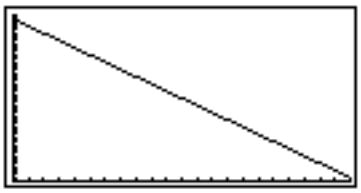
Producing the table and exploring it reveals the following.

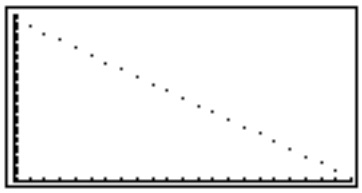




This reopens the question of what is possible and what is not? Most would say 22 adults and one child is fine, but is it? This will help to develop an understanding that when algebraic rules are used to model situations, restrictions may be required so that a sensible model results.

Students can also produce a graphical display of their findings as a nice alternative. Two options are:





The first was produced by G-CON and the other by G-PLT.

Again fruitful discussion can be had as to which of these are appropriate for the situation we have. Most students at an early age seem to believe the line is an appropriate graph in this situation.

This activity can be modified in a number of ways and can also be followed up with more patterning type exercises. You might like to extend the students to cases where the summation of pattern members is required (ie. arithmetic or geometric series). Most students in lower secondary are quite capable of solving senior type problems of this nature if they have the support of a graphics calculator or the like. Hence student's ability to think and form models is developed and they enjoy the feeling of having solved a 'meaty' problem.