Parabolic motion reborn

Level
Upper secondary

Mathematical Ideas
Parametric equations and the application of mathematics

Description and Rationale
This lesson leads students to compare the properties of parabolic motion when the initial conditions of flight are varied. The graphics calculator provides an efficient medium to simulate projectile motion once the students understand the basic concepts of projectile motion. This is a very powerful feature and allows students to see what is happening and get a genuine feel of the concept of time of flight. It also offers tools to help solve problems that have traditionally solved by mind, paper and pen. It is left to the reader to consider the value of this approach – mind and calculator.

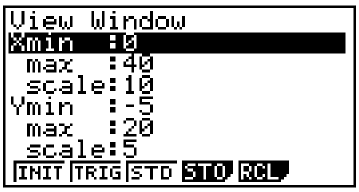
Students can investigate how the height above ground at a given time, maximum height reached and range of the object depend on the angle of projection.
Consider a projectile that is launched at 20 metres per second at 30, 45 and 60 degrees above the horizontal.

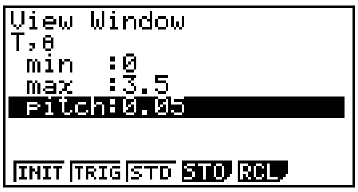
In the first of these cases the students should find that the components of the motion can be described by the pair of parametric equations, $v_h = 10t\sqrt{3}$ and $v_v = 10t - 5t^2$

Now, in graph mode the students can simulate the motion and leave a trail for analysis.

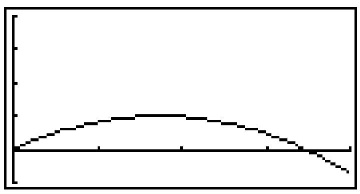
Once the graph type is changed to parametric (using TYPE (F3)) the equations can be entered.

Appropriate settings for the view window are seen below.

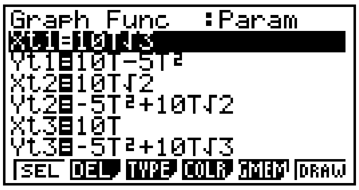


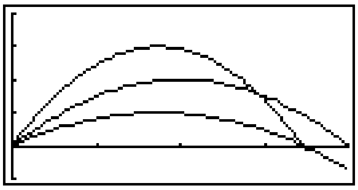


Drawing the graph (using DRAW (F6)) reveals the path seen opposite.

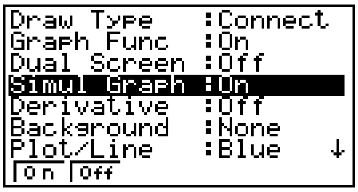


Entering the other two sets of parametric equations that describe the paths being investigated will result in the graph seen below.

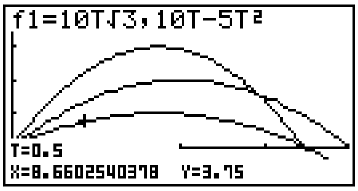


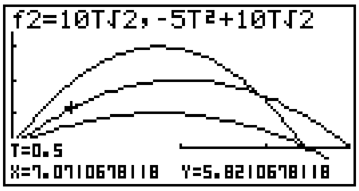


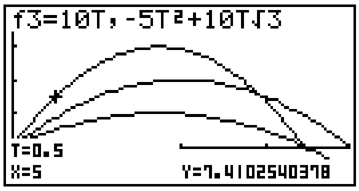
If you enter the SET UP of the graph mode (SHIFT then MENU) and turn the Simul Graph option on, the projectiles will be launched in unison. This provides the viewer with an appreciation of the time of flight of the projectiles. This is a great advantage to students in their quest to understand projectile motion.



While the graph is on screen, students can use the TRACE (F1) facility to explore the path and compare the features of each path. Using the up and down arrow will allow you to swap between paths.



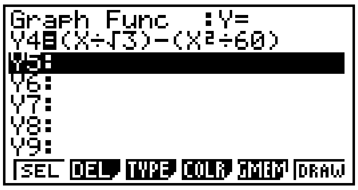


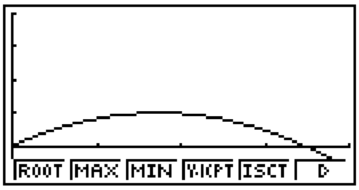


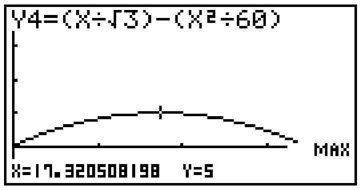
Note how the cursor simulates the projectile and you can see where each object was a 0.5 of a second after launch. Also given are the horizontal and vertical positions at this instance.

Students can tabulate values of interest for comparison purposes. It may or may not be possible to trace to the peak of flight and point of impact with the present settings for time (T). The challenge still exists for the students to find these points. However, one may ask whether the level of accuracy we once required is important. It is left to the reader to consider this. But, please read on.

When drawing the graph of a set of parametric equations the treasure trove of tools in the G-Solv (F5) menu does not work. Students could, however, substitute to get the vertical height in terms of the horizontal distance travelled, change the TYPE of graph to a Y=, and then use the MAX and ROOT within G-Solv tools as seen in the screens below.







Many possible investigations can now take place. Students could investigate the effects of changing the launch height and launch speed.

In addition to this students may use the calculator as a tool to aid in the solution to problems like:

> *A plane flying at 300m above the ground at 150 metres per second releases a food parcel at a point that is 800m (horizontally) from the target. Did the pilot calculate well? If not determine the point at which the parcel should have been released.*

It is left to the reader to consider whether or not the use of the calculator to aid in the solution of such problems is desirable.