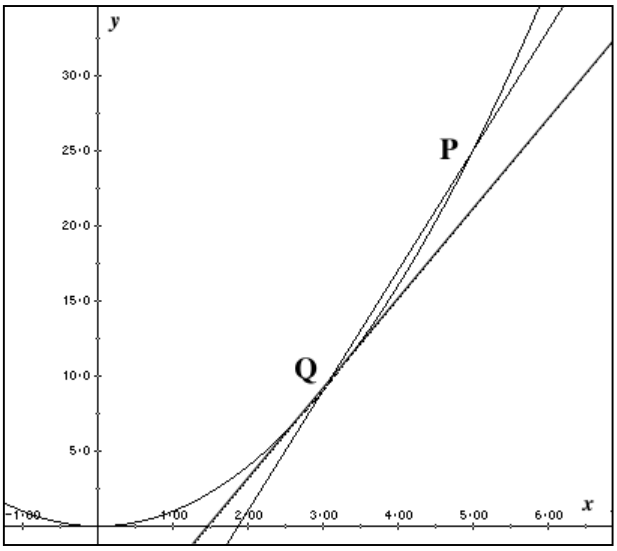
Slope of a tangent

Level
Upper secondary

Mathematical Ideas
Slope of a tangent, limits

Description and Rationale
This lesson acts as an introduction to the concept of a limit and differential calculus. The graphics calculator allows students to perform and *display* repeated calculations with ease, which is of great advantage in this task.



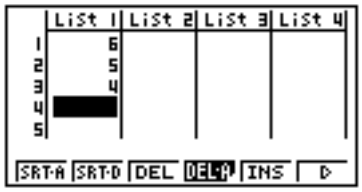
Students are asked to consider the problem of determining the slope of the tangent to the curve $y = x^2$ at $x = 3$. They would be aware that they can simply determine the slope of any chord (like the slope of PQ) on this curve, but the tangent is another matter.

Once the students appreciate that we could consider P as a moving point and move it toward the fixed point Q they will see that we may have a way to approximate the slope of the tangent. That is, make the chord become as close to a tangent as possible. Dynamic geometry packages offer the lovely option of being able to 'drag' the chord toward the point of tangency.

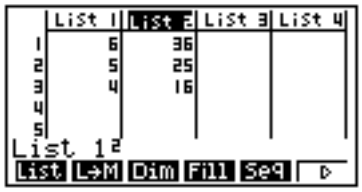
Students should now produce a table, using mind, paper and pen, illustrating the slopes of the chord at 3 different points along the curve as P approaches Q. For example when $x = 6.5$ and 4. They should be encouraged to consider the situation when P maps to Q.

We can use the graphics calculator to efficiently investigate how the slope of chord PQ varies as P moves closer to Q at many more points than four and at much smaller increments than would otherwise be engaging and hence, see whether or not a reasonable approximation of the slope of the tangent at Q can be found. The lists in the calculator can be used to do this.

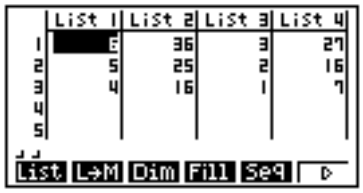
In LIST or STAT mode we can type in the required values.

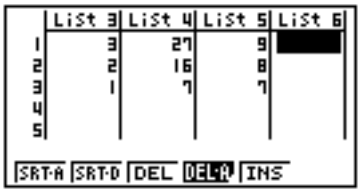


In List 2 we can calculate the value x^2 as shown opposite. This is completed by highlighting the list name, selecting OPTN and LIST (F1) and then squaring the values in list 1 as shown.



In List 3 we can calculate the *x* step (or run), in List 4 the *y* step (or rise) and then in List 5 the slope of the respective chords as seen below.

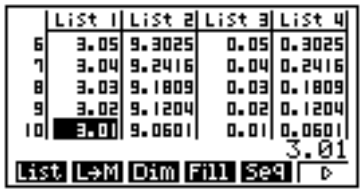




This should replicate what the students have done with mind, paper and pen. It acts as a nice check and simple way to get started.

Now students can begin to experiment with smaller increments between chords, and chords that are closer to being tangential.

The screens below show one possible output.



Care will be required when determining how many elements are generated in List 1. It has a limit of 255 elements and the calculator will return a memory error if you try to exceed this.

Discussion can then follow this activity about the value of the slope of the tangent that seems sensible and the concept of a limiting value.