Exploring derivatives of trigonometric functions

Level
Upper secondary

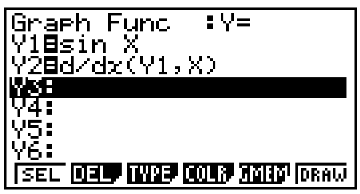
Mathematical Ideas
Trigonometric functions, graphical representations, derivatives

Description and Rationale
The graphics calculator is an ideal device for the structured investigation of patterns in mathematics. One of the more difficult concepts in many mathematics programs involve the relationships between functions and their derivatives. Students often gain a better understanding of these relationships through pictorial representations. The example shown below uses a “black box” approach to the determination of the derivative. This is a valid approach only once students are familiar with the basic definitions and interpretations involved in determining the derivative of a function from first principles. While this example involves trigonometric functions the general approach could be used with a variety of functions.

This structured investigation is designed for students who are not aware of the rules for the derivatives of y = Asin kx and y = Acos kx or who may need to revisit these concepts.

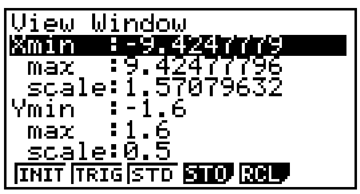
The calculator is set so that a function and its derivative are plotted simultaneously. Color can be used quite powerfully to distinguish between the function and its derivative. In the example below sin$x$ has placed in Y1, while the commands stated below define Y2 to always be the derivative of Y1.

To define the derivative the OPTN button then the CALC (F2) menu is used. The defining of the function Y1 is via the VARS then GRPH (F4) menu, the x defined in the bracket is the variable that the derivative is being found with respect to.

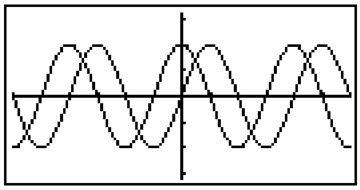


The view-window can be set for trigonometric functions using the menu option of TRIG, the minimum and maximum Y-values will need to be defined. The angle measure must be in radians for the screen capture shown. (see SET UP if degree values appear).

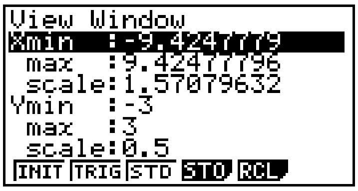
The view-window can be accessed via the function buttons below the screen.

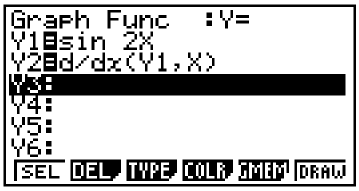


The graphs of $y = \sin x$ and its derivative suggest that the derivative is a cosine function, its amplitude is the same as the original function and its period is also the same as the original function. From this the derivative appears to be $\cos x$.



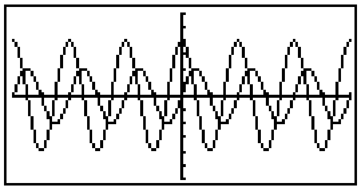
The advantage of the set up used here is that as the function in Y1 is altered, the function in Y2 will always the derivative of Y1. After altering the view window Y1 has been altered to sin 2x and the resulting graphs produced.





Students would be expected to notice that the derivative function is:

- (i) cosine in nature
- (ii) of the same period as the original function
- (iii) twice the amplitude of the original function



This leads to the conclusion that $\frac{d}{dx}(\sin 2x) = 2\cos 2x$

Students would then be asked to suggest a rule and test it using y = sin 3x and from this investigation make a general statement about the derivative of sin kx.

i.e. $\frac{d}{dx}(\sin kx) = k\cos kx$

Students would then complete similar investigations to determine the general forms:

$$\frac{d}{dx}(A\sin kx) = Ak\cos kx$$

$$\frac{d}{dx}(A\cos kx) = -Ak\sin kx$$