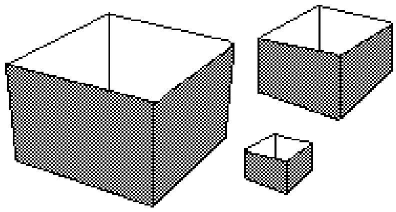
13. Extending the Open Box Problem (MM3/4) (KP:2.2;5.3;7.1;9.1;9.3;9.4)

An open box is constructed by cutting equal size squares from the corners of a rectangular sheet of cardboard and folding up the sides. Your initial task is to make the cuts so that the box has maximum volume.

Then you will experiment with various sized sheets of cardboard, in which the breadth is always twice the width, to see how the maximum volume varies with the width.



1. Start with a cardboard sheet with dimensions 5 cm by 10 cm. Construct a function which will give the volume V (cm^3) of the box as function of the length x (cm) of the side of the square cut.

2. Use the graph of the function in question 1 to approximate the maximum volume to the nearest cubic mm. Also give the approximate length of the side of the square cut which produces that volume.

3. In order to investigate what effect changes in the dimensions have, repeat questions 1 and 2 for cardboard sheets which are 10 cm by 20 cm, 15 cm by 30 cm, and 20 cm by 40 cm. Collect your results for all 4 in a table that displays breadth of the sheet, width of the sheet, approximate maximum volume and corresponding length of square cut.

4. How does the size of the cut in each case seem to compare to the width of the cardboard sheet? State the comparison verbally and as a mathematical statement.

5. Experiment to see if the maximum volume is directly proportional to the width, the square of the width, or the cube of the width of the cardboard sheet. If you find a relationship, express it verbally and as a mathematical statement.

6. Justify your answer to question 5 using a calculus approach.

Note: Like the fencing problem, this tasks, extends a standard maximum / minimum problem to consider a possible generalisation. Using the graphics calculator, students solve the problem of maximum volume, and apply a curve-fitting approach to generalise their solution.