17. The Days are Getting Longer (MM3/4) (KP:2.2;4.3;4.4;5.3;7.7;11.1)

The number (y hours) of daylight in Melbourne x days after the longest day of the previous year can be modelled by the rule

$$y = 12.125 + 2.625 \cos \frac{2 \quad x}{365}$$

1. Find a value of x for which the length of the day is increasing fastest, and find out how much the length of the day is changing at this time.

2. This model is based on the number of days elapsed since the longest day. If you wanted it to use the number of days elapsed since the start of 1996, how would the function change? (The longest day of the year in 1995 was the 22nd of December).

3. Using the information from your answer to question 2, predict the dates and number of daylight hours of the following significant events:

(a) Summer Solstice;
(b) Winter Solstice;
(c) Spring Equinox;
(d) Autumn Equinox.

Check your answers against the dates listed for the above events in the 1996 calendar.

Note: This task provides a context for examining the derivative function and possibly the second derivative function. Often, the graphs of a function and its derivative function are not easy to view simultaneously due to scaling problems. In the task above, the graph of the derivative function can be explored independently from the graph of the original function to tackle the problem at hand. Alternatively, a table of values of the function and its derivative function can be used to locate the points of interest. The second derivative may be a useful additional feature in determining the key points.