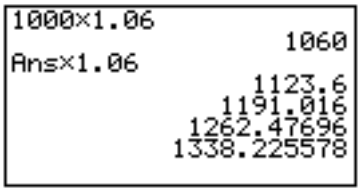
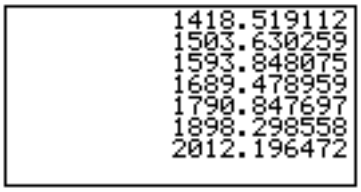
30. Compound Interest (GM)

(KP:2.1;2.2;3.1;5.1;5.3;7.1;11.1)

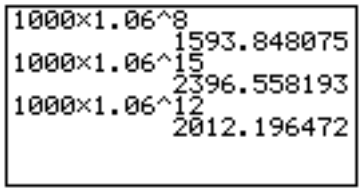
Huynh invests $1000 with a bank that calculates interest at a rate of 6% per year, credited at the completion of each year. After how many years will he have doubled his money? (To the nearest year)

Approach 1 - Repeated calculation

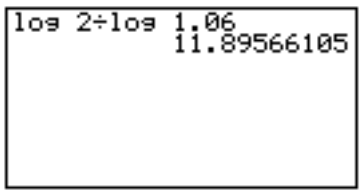




Approach 2- Guess, check, & improve



Approach 3 - Using manipulation



When does $A = 2P = PR^n$ -> $2 = R^n$ ->
$\log 2 = n \log R$ -> $n = \log 2 / \log R$

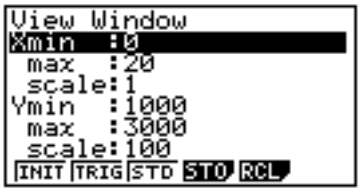
Establish that $n = \frac{\log 2}{\log 1.06}$ and then round up.

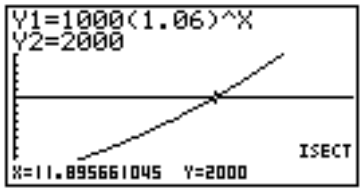
Approach 4 - Function Rule and Table

Set start value at 0 and pitch value at 1, and calculate which year.

Approach 5 - Function Rule & Graph

Enter Y1 = 1000 $(1.06)^x$ and Y2 = 2000
Alternative - plot Y2 = 2000, Y3 = Y1 - Y2 by Trace or Jump-to feature.





Extension: Doubling Time

A rule of thumb for the number of years, d, to double your money, is: $d = \frac{70}{\text{interest rate}} = \frac{70}{r}$ where r is the interest rate expressed as a percentage.Complete the table to verify the accuracy of this rule of thumb for various values of r. Try to calculate the doubling time (rounded up to the nearest year). Is it a reasonable estimate for the usual range of interest rates?

Interest Rate (r)	**4**	**8**	**12**	**16**	**20**	**25**
Doubling Time (from Graph or Table)						
Doubling Time (from d = 70/r)						

Why does the rule of thumb work? Approach 3 shows that doubling time is $\frac{\log 2}{\log (1 + \frac{r}{100})}$. Now log 2 0.6931 or roughly 0.7. So the doubling time is roughly $0.7 / \log(1 + \frac{r}{100})$; and the rule of thumb can be written $0.7 / (\frac{r}{100})$.

Graph $y = \frac{r}{100}$ and $y = \log(1 + \frac{r}{100})$ for 0 r 30 on the same set of axes to see how close they are.

Note: The purpose of this task is to illustrate a number of methods by which a standard application involving compound interest can be approached. The method used may depend on the entry level of the students, but each method has intrinsic merit. This task also showcases some key calculator features.