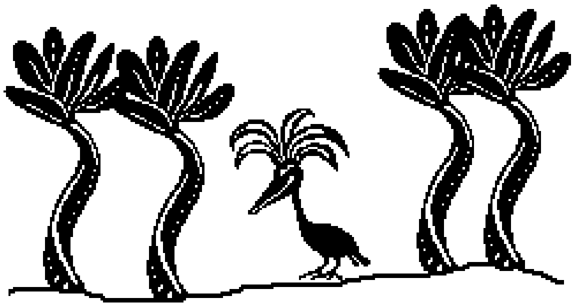
42. The Old Swamp (FM) (KP:12.1)

An endangered species of bird, the 'Yellow-Crested Farnarkler', is found to live only in and around a certain swamp known as the Old Swamp. The birds were counted in 1986 and the population was found to be 125. A year later the population had increased to 150.



1. Suppose that the populations in successive years formed an arithmetic sequence. What would be the 1996 population in this case?

2. Now suppose that the populations in successive years followed a geometric sequence. What would be the 1996 population in this case?

Eventually the number of Farnarklers in the Old Swamp builds up to 350. Scientists who are monitoring the situation then consider it safe to resettle another swamp by transporting 30 Farnarklers per year from the Old Swamp to 'New Swamp'. The initial Farnarkler populations of the Old and New Swamps are thus 320 and 30 respectively, and it is assumed that each population increases by 15 percent per year.

3. Find the Farnarkler population of each swamp after one year, just after the second set of birds has been transported.

4. The size of the Farnarkler population at the Old Swamp after n years, P_n, just after the birds have been transported for that year, is given by the difference equation

$$P_n = 1.15P_{n-1} - 30, \quad n = 1, 2, \ldots, \quad P_0 = 320$$

Write down a similar difference equation for the size of the Farnarkler population at the New Swamp after n years, Q_n .

5. (a) Predict the Farnarkler populations of both the Old Swamp and the New Swamp after five years.

(b) Predict the number of years, to the nearest whole number, that pass before the Farnarkler populations of the two swamps are most nearly equal.

Note: Arithmetic and geometric sequences can be investigated on a graphics calculator using sequence mode. In this task, adapted from an early VCE CAT question, students explore the properties of such sequences and their usefulness for prediction in this problem context.