# 43. Fiddling with Fibonacci (FM) (KP:12.1)

The Italian monk Leonardo de Pisa (late $12^{th}$ and early $13^{th}$ centuries), who was also known as Fibonacci, is famous for the sequence named after him. The Fibonacci sequence is defined as follows: let the first two terms be 1 and 1 and take each subsequent term to be the sum of the two preceding terms.

**1.** Write down a difference equation for the $n^{th}$ Fibonacci number $f_n$ in terms of $f_{n-1}$ and $f_{n-2}$.

**2.** Enter the Fibonacci sequence into your graphics calculator and find $f_{20}$ both graphically and via a table of values.

**3.** Explore, for a number of values of n, the expression $f_n f_{n-2} - f_{n-1}^2$. What does this suggest? Verify the result for the first 20 Fibonacci numbers.

**4.** Explore, for a number of values of n, the expression $f_n / f_{n-1}$. What does this suggest? Verify the result using a graphical and tabular method.

**5.** The French mathematician Lucas (late $19^{th}$ century) looked at a Fibonacci-like sequence in which the generating rule is the same but the first two terms are 1 and 3. Let $l_n$ be the $n^{th}$ Lucas number.

(a) Enter the Lucas sequence into your graphics calculator and find $l_{20}$. How does it compare with $f_{20}$?

(b) Determine whether the Lucas numbers possess the same properties as the Fibonacci numbers that you found in Questions 3 and 4.

Note: This activity involves an investigation into Fibonacci-like sequences and exploits the sequence mode and list variable features of the graphics calculator.