Simple Annuities – Future Value.

OBJECTIVES

(i) To understand the underlying principle of a future value annuity.

(ii) To use a CASIO *CFX-9850GB PLUS* to efficiently compute values associated with future value annuities.

Note that we suggest you complete the activity called Simple Annuities – Present Value before starting this set of activities.

EXPLORATORY ACTIVITIES

Activity 1: A simple investment annuity
One way to invest money is to *deposit an amount of money into an account at regular time intervals.*

For example, I could deposit $100 (**regular payment**) into an account every month. The bank would then pay me **interest** in a similar way to compound interest, paying me a % of the balance of the account at a certain point in time (say every month). This interest is added to the account and so the account grows in two ways, by me adding an amount every month and by the bank adding interest every month too.

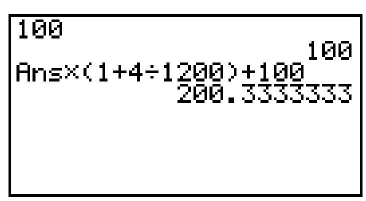
This form of investment is called a *simple future value annuity* as the value of the annuity is in the future not the present.

Suppose I pay $100 each month into a simple annuity for which the bank pays interest of 4% per annum compounded monthly.

Important note. *In most instances, the bank does not pay interest for the first compounding period (one month in this case). Also the interest is paid at the end of the compounding period paid on the value of the annuity at the end of the previous period.*

To determine the value of the annuity at the end of each month we perform a repetitive calculation as follows:

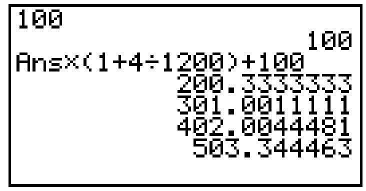
- Enter RUN mode
- Enter 100 and commit it to the calculators answer mode by pressing EXE.
- Multiply the Ans(wer) (SHIFT then (-)) by $(1+\frac{4}{1200})$ and add 100 and calculate the answer by pressing EXE



You should find that the value of the annuity is $200.33 (to the nearest cent).

Note that this is the **future value** of the annuity at the end of the second month; at the end of the first it will be worth just the $100 you initially invested.

At the end of three months you should find that the value of the annuity is $301.00 (to the nearest cent) and so on.



Activity 2: Generalising a simple investment annuity.
A formula exists for computing values associated with Present Value annuities. Its use saves us from repeatedly pressing the equal sign on the calculator.

The derivation of the formula relies on knowledge of geometric series. You might like to research this.

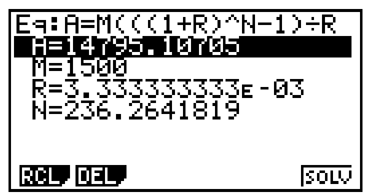
Firstly, define each quantity as follows:

- Let the future value of the annuity (final balance), after *n* compounding periods, be *A*
- Let the regular contributions be *M*
- Let the percentage interest rate per compounding period be *r* (expressed as a decimal)
- Let the number of compounding periods be *n*

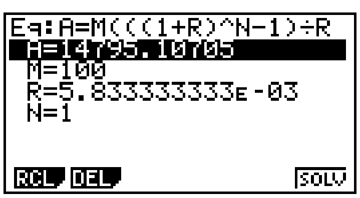
The formula is:

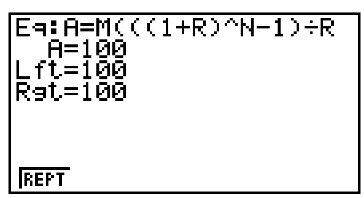
$$A = M\left(\frac{(1+r)^n - 1}{r}\right)$$

The *9850Gb PLUS* can be use to compute the value of *A*, *M, r or n if all but one of the variables is know.* In EQUA mode, after choosing SOLV(er) (F3) the formula can be entered using bracket carefully.



So, if $M = 100$, $r = R = \dfrac{7}{1200}$, and $n = N = 1$, you should find that $P = 100$, can you explain why?





Once the result is given you can continue to use the formula you have entered. Simply press REPT (F1) and you will be prompted for the values of the variables again.

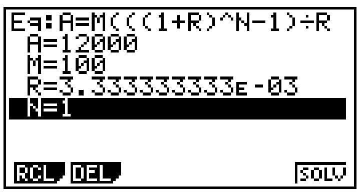
This is a useful feature if you have many of the same computations to do where the values of the variable change often.

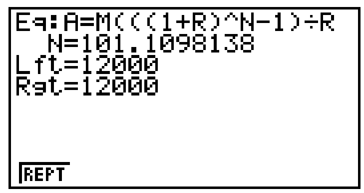
Activity 3: Using the formula more broadly.
Of course we can use the formula for more than just computing the future value of the annuity after *n* interest periods.

Suppose we wanted to know how long we would need to invest $100 per month in a simple annuity to grow it to $12000 (so we could buy that Sharp LCD panel) if the interest paid was 4% compounded monthly.

This would require us to solve the equation $12000 = 100\left(\dfrac{\left(1+\dfrac{4}{1200}\right)^{n}-1}{\dfrac{4}{1200}}\right)$ for *n*.

We could proceed as follows:





How long? I wonder what the price of the Sharp LCD screen would be by then, more or less? Why?

EXERCISES
The purpose of the exercises is to give you an opportunity to do some independent work with simple annuities and to develop fluency with both the mathematical concepts and Casio *9850GB PLUS*.

Exercise 1.
Jillian has just turned 16 years old and has her mind set on buying a new car by the time she is 20 years old. To achieve this goal she decides to save as much money as she can over this period. She can afford to save $440 per month. She regularly places the money into an annuity that pays 5.5% per annum compounded monthly.

a) Show that she will have $ 23 563.25 to spend.
b) Determine the amount of interest she has earned over this time.

Exercise 2.
Lucy can invest $150 per month in an annuity for 2 years. What interest rate must she invest at if the annuity is to be worth $5 000 at the end of the 2 years. Assume the interest is compounded monthly.

SOLUTIONS

Exercise 1
b) $2443.25

Exercise 2
Approximately 32.9% per annum. (I wonder if this will ever be possible?)