Understanding and working with compound interest.

OBJECTIVES

(i) To understand the underlying principle of compound interest

(ii) To develop a formula for compound interest

(iii) To use a calculator to efficiently compute values associated with compound interest situations

EXPLORATORY ACTIVITIES

Activity 1: Growing money in a compounding way.
When someone *invests* money (an investor), they are essentially loaning money to a bank or similar institution. The bank then uses that money for some purpose and must pay the investor for the right to use it. The payment is often termed *interest*.

One common way interest is paid is as follows:

- Investor places a set amount of money (the **starting principal**) into an account
- Bank agrees to pay a set percentage of the investment (the **interest**) at regular intervals (the **interest period**). The set percentage is called the **interest rate.**
- The **interest periods** might be monthly, yearly or whatever is agreed to.
- When paid the interest is added to the starting principal to form a new starting principal for the next interest period.
- **The process then repeats**

This type of process is called ***compound interest*** because the interest is added on to the principal and compounds the investment.

Note that the amount of interest will increase for each period from the previous due to the addition to the principal.

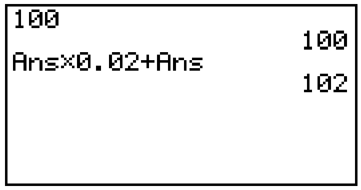
One definition of the word compound is *to intensify by adding new elements*.

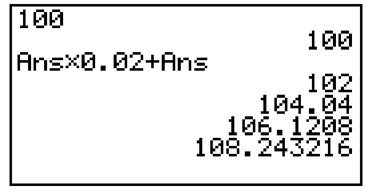
Lets consider the following example. Suppose I invest $100 for 2 years and the bank states it will pay me 2% interest, compounded every month for the two years.

The **starting principal** is $100, the **interest period** is one month, and the **interest rate** is 2%. What is the new starting principal at the end of the *each* month?

To calculate this we can multiply 100 by 0.02 and then add this result to 100 and then repeat the process. To do this repeatedly we can perform the following operations on the Casio CFX 9850GB PLUS:

- Enter RUN mode
- Enter 100
- Commit it to the calculators answer mode by pressing EXE
- Multiply the answer by 0.02 and add the answer, the ANS key will help (SHIFT then (-) and press EXE.
- Repeat the process by pressing EXE repeatedly





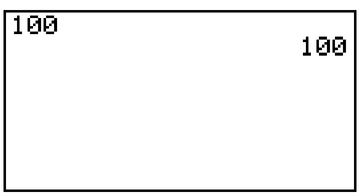
Doing this should return the following 'new' principals: 102, 104.04, 106.1208, 102.243216 and so on. Of course some rounding is necessary.

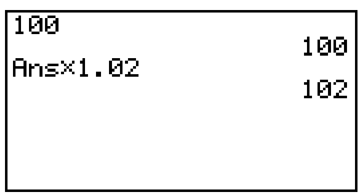
Activity 2: Computing it differently.
Instead of *multiplying 100 by 0.02 and then adding this result to 100* and then repeating the process we could do the following:

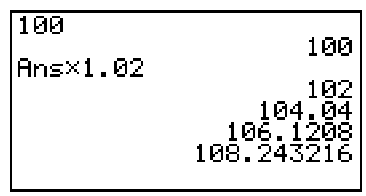
Multiply 100 by 1.02 (increasing 100 by 2%) and then multiply this result by 1.02 and continue in this way.

To do this repetitively on the calculator, do the following:

- Enter 100
- Commit it to the calculators answer mode by pressing EXE
- Multiply the answer by 1.02 and press EXE
- Repeat the process by continually pressing EXE







Note that the repeated multiplication is the same as finding a power.

Activity 3: Generalising the process.
The last way (seen in Activity 2) of computing the new principal for a compound interest calculation can be *generalized*, **that is writing as a formula**.

Firstly, define each quantity as follows:

- Let the final balance, after *n* compounding periods, be *A*
- Let the initial quantity be *P*
- Let the percentage interest rate per compounding period be *r* (expressed as a decimal)
- Let the number of compounding periods be *n*,

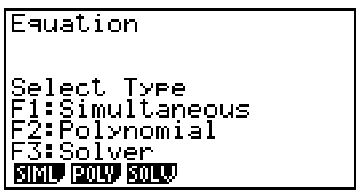
Now we can write:

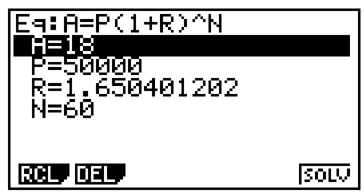
$$A = P(1+r)^n$$

The *9850GB PLUS* can be use to compute the value of A, P, r or n if all but one of them are known.

Using EQUA mode the formula can be entered as $A = P(1+R)^N$ by doing the following:

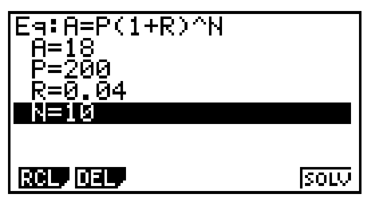
- Enter EQUA mode
- Select the SOLV(er) option (F3).
- Press the red ALPHA key first to be able to enter the red letters on the key pad
- The = sign can entered by pressing SHIFT then (-)



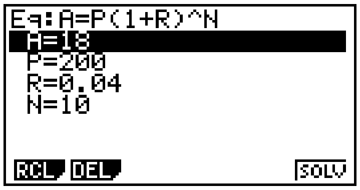


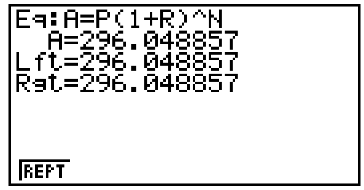
You can now see that each variable is listed below the formula. Each can be changed to the values you require.

Suppose $P = 200$, $r = R = 0.04$, and $n = N = 10$. Be to press EXE between each change that is made so it is accepted by the machine.



Now to find the value of *A* simply put the cursor on *A* and SOLV(e) (F6).

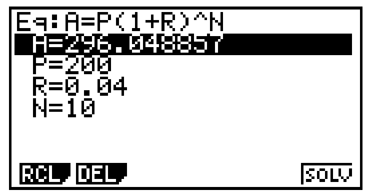




You should find that $A = 296.05$ correct to two decimal places.

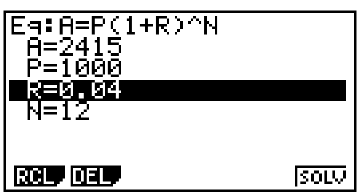
The LfT and Rgt values are the value of the left and right sides of the formula if the values of each variable found/given are substituted into the formula – they should be equal. This acts as a check to show the machine has computed correctly.

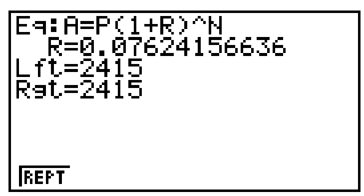
Once the result is given you can continue to use the formula you have entered. Simply use REPT (F1) and you will be prompted for the values of the variables again.



Activity 4: Using the generalization more broadly.
Of course we can use the formula to do more than just compute the principal after *n* interest periods.

Suppose we wanted to know the interest rate required such that an initial investment of $1000 dollars grew to $2415 in 12 compound interest periods. This would require us to solve the equation $2415 = 1000(1+r)^{12}$ for *r* an can be done as follows:





Note that the value of R was not changed from last time and the cursor is placed on it to finds its new value.

Activity 5: How do the banks speak?
The banks generally talk in terms of *per annum* interest rates.

For example, they might say,

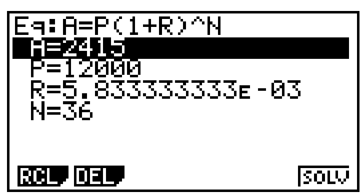
"Invest now, earn 7% per annum compounded monthly."

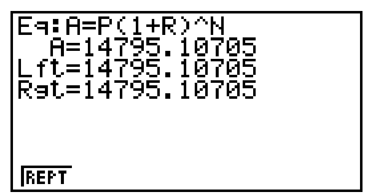
This means that they will add $\frac{7}{12}$ of a percent of the principal at the start of each month to that principal to determine the new starting principal.

Suppose I invested $12 000 for 3 years under the conditions above. To find out how much I had at the end I could compute:

$$12000(1+\frac{7}{1200})^{36}$$

Note that we need to realize that there are 36 interest periods not 3!
To enter a fraction, like $\frac{7}{1200}$, in the *9850GB PLUS*, simply enter 7 ÷ 1200 for R in EQUA and press EXE.





You should find that the $12 000 had grown to $14 795.11 (to the nearest cent).

EXERCISES

The purpose of the exercises is to give you an opportunity to do some independent work and to develop fluency with both the mathematical ideas and processes and the use of the CASIO *9850 GB PLUS*.

Exercise 1.
Check that an investment of $5600, invested at 7.2% per annum compounded monthly, has a value of $8018.02 after five years.

Exercise 2.
Compare the values of an investment of $5600 after 12 months if it earns 7.2% compounded:

- i) annually
- ii) quarterly
- iii) monthly
- iv) fortnightly

Exercise 3.
Kellie invested $5600 at 7.2% pa compounded quarterly. What was the value of her investment after four years?

Exercise 4.
Simon invested $5600 at 7.2% pa compounded semi-annually (i.e. twice every year). Predict whether his investment would accumulate, in four years, to more or to less than that of Kellie in the previous question. Use your calculator to check your prediction.

SOLUTIONS

Exercise 2
i) $6003.20 ii) $6014.22 iii) $6016.78 iv) $6017.47

Exercise 3
$7449.93

Exercise 4
$7431.32