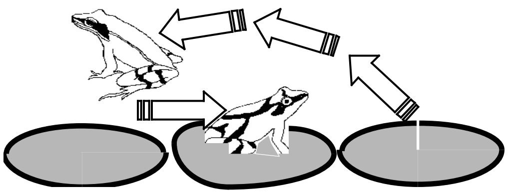
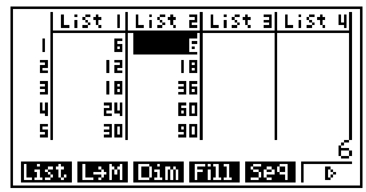
Describing and predicting the behaviour of systems

A unique learning experience – Some assessment ideas





A product of the Noel Baker Centre for School Mathematics

WIP (Work in progress)

LUMAT-NSW (2003) is the initiative of the

Noel Baker Centre for School Mathematics and CASIO AUSTRALIA.

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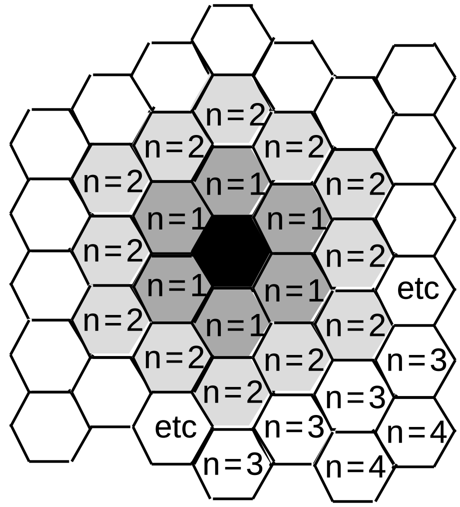
A bee of a problem

Examine the piece of honeycomb shown at right. If we start with any central cell, then note that it is surrounded by a ring of cells (labelled n=1). Let us call this darker shaded chain of hexagons Ring 1. This in turn is ringed by a larger chain of hexagons (lightly shaded and labelled n=2) which we shall call Ring 2. Similarly Rings 3, 4, 5, 6 etc exist.

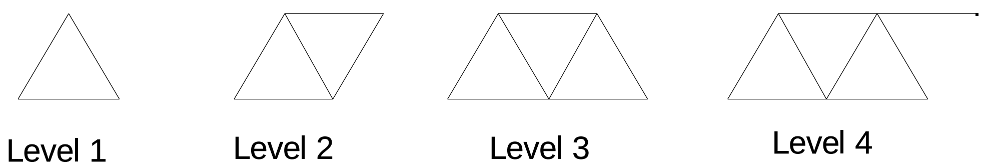


Investigate the number of hexagonal cells in each ring. Find formula to represent the pattern of results? If possible, prove that your formula is true in general.

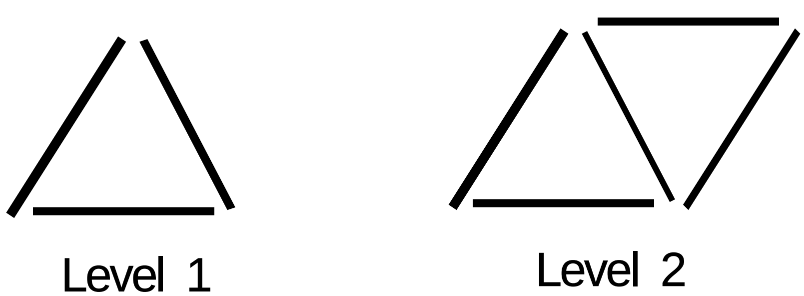
If each cell holds 1 unit of honey, how much honey is in rings 1 to 50?

Chains of polygons

Consider the following sequence of triangle chains. Each chain is considered to be a different **level** of the chain. The first few levels are shown in this diagram.



If you were asked to build these structures with toothpicks, it would take 3 toothpicks for level 1, 5 toothpicks for level 2, and so on.

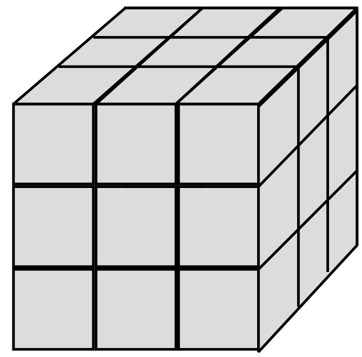


Investigate the pattern in the number of toothpicks needed at each level, making sure to

- document the results of your investigations appropriately
- describe how the pattern develops
- find a general formula to describe the pattern
- use the structure of the chains to prove that your formula is correct
- would 2506 stick make a 'complete' chain?

Cubes in a cube

Find the formula for N, the number of cubes within an $n \times n \times n$ cube. [Hint: do the totals all seem to be squares? … and, if so, of what special numbers?]



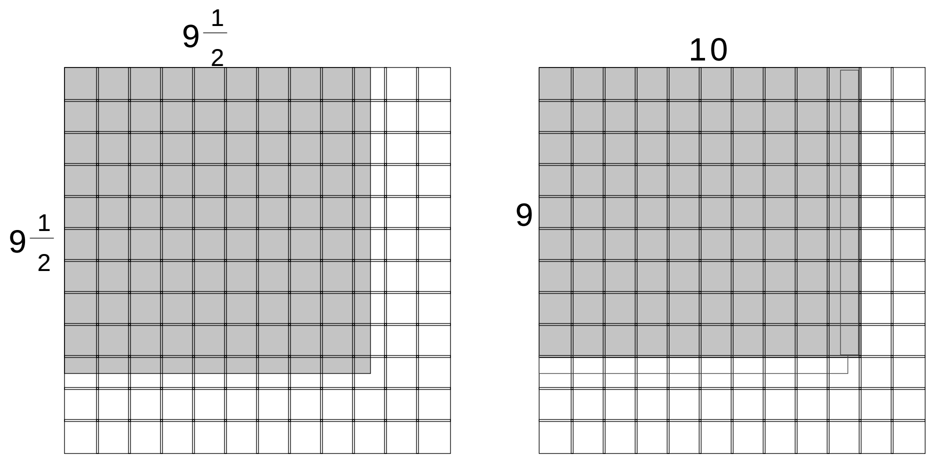
Ananarf squared

1. Is Jo's father's method correct?

2. Check to see if the method works for $\left(19\frac{1}{2}\right)^2, \left(29\frac{1}{2}\right)^2, \left(99\frac{1}{2}\right)^2, \left(999\frac{1}{2}\right)^2 \ldots$

3. Does the method work any differently for numbers that do not end in 9? (e.g.)

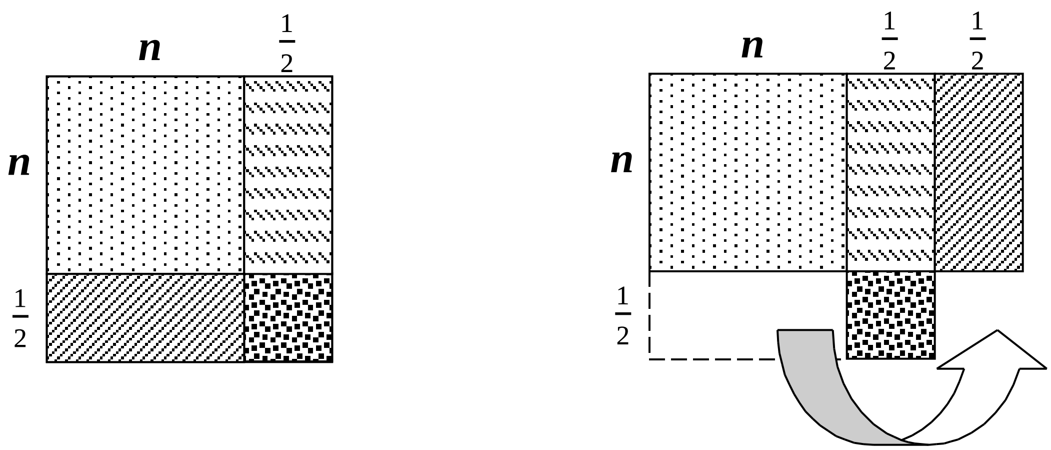
4. Jo says that her father had drawn these diagrams to show that the method works. Can you see any flaws in his thinking?



5. Make a conjecture for the value of a-number-"ananarf" squared - write it in words.

6. Make a conjecture for the value of - write it as a formula.

7. Prove your conjecture with the help of one of these diagrams



6. What equivalent result is produced by the other diagram?

All Baker Centre work is considered as a WIP

Some notes for some of these problems

Cubes in a cube

The 3 × 3 × 3 cube shown contains $3^3 = 27$ unit cubes, $2^3=8$ 2 × 2 × 2 cubes and $1^3=1$ 3 × 3 × 3 cube.
Hence this 3 × 3 × 3 cube contains

$$1^3 + 2^3 + 3^3 = 1 + 8 + 27 = 36$$

cubes of any size.

An $n \times n \times n$ cube contains n^3 unit cubes, $(n - 1)^3$ 2 × 2 × 2 cubes etc, with the total number of cubes of any size being
Students may be able to see the link between the sum of cubes and the square of the sum

$$1^3 + 2^3 + 3^3 + \ldots + n^3 = \left[n \times \frac{(n+1)}{2} \right]^2$$

$$1^3 + 2^3 + 3^3 + \ldots + n^3 = (1 + 2 + 3 + \ldots + n)^2$$

of the first n natural numbers, particularly if they notice the pattern of squares.

Ananarf squared

This is a "shortcut" that students will like to use to show off to those who don't know it.
Note that representing n^2 by the area of a rectangle is by no means an obvious technique to most students.

