'Analyzing the Annulus' Investigation

A note to teachers:

This is a 'quirky algebraic modelling' investigation. That is to say a 'quirky' problem, rather than 'real world' problem, generates the model.
It is important to keep in mind the purposes of running algebraic modelling investigations such as this given the solution has negligible 'real world' importance.
Depending on student competence this activity may be run in several ways. Two of these are: as a student-directed lesson by issuing pages 2-5 for students to work through, or as a whole-class, teacher-directed activity, issuing just page 2.

The purposes of this activity include:

- To engage students mathematically

- To give students experience of applying algebra to a problem

- To efficiently generate a table of values:
  - To enable students to 'see the problem' represented in the numbers
  - To expose students to finding a solution within a table

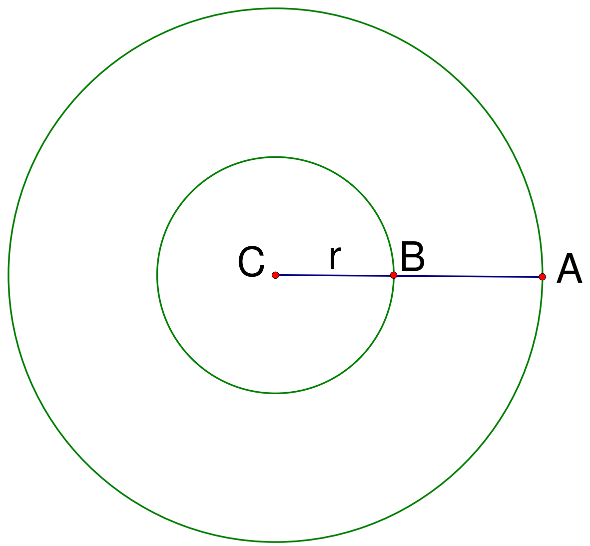
- To efficiently generate graph/s for the problem:
  - To enable students to 'see the problem' represented in the graph/s
  - To find a graphical solution

(And possibly the most important purpose of this activity)

- To create numerous opportunities for the teacher to ask 'Working Mathematically' questions during the activity, thereby giving students many opportunities to think mathematically, make mental links and gain conceptual understanding.

NOTE: If you are new to TABLE and GRAPH modes you may find it beneficial to first work through the worksheet 'Self-Guided_9860_TABLE-GRAPH'.

Algebraic Modelling Activity Analysing the Annulus



CA = 40cm

In the diagram above, point B moves along CA to vary the size of the anulus. The radius of the smaller circle is labelled r. The length of CA is 40cm.

The Challenge:

What does the radius of the small circle (r) need to be in order for the area of the annulus to equal the area of the small circle?

1) Using Y for area and 40 for radius find an expression for the area of the large circle. Write this here and enter it in line 1 of TABLE mode ____

2) Using Y for area and X for radius (use X,θ,T for X) find an expression for the area of:
   a. The small circle ____________________________ Enter this in line 2
   b. The annulus ____________________________ Enter this into line 3

3) Generate a table of values for all 3 expressions using X = 0 to 40, Step = 1 using SET (F5).

   a. Why are all the Y1 values the same?

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   ____
   ____

   b. Explain why the Y2 values change as they do.

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   ____
   ____

   c. Explain why the Y3 values change as they do.

   ____
   ____
   ____
   ____

d. Do the Y2 and Y3 values seem to be linked to the Y 1 values? Explain your answer.

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e. Attempt to find an approximate solution to the problem from the table. Write it here.

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4) Considering the values from the table use V-Window to set up appropriate axes for the graphs of these expressions. Then go to SET UP and ensure Axes and Coordinates are turned On

5) Go to Graph mode (MENU then 5). Select only the Y1 graph (using F1). Now DRAW (F6)
What part of the problem does the Y1 expression describe? Trace the graph. Explain the shape of the graph.

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6) EXIT, de-select Y1 and select Y2. Then DRAW. Turn the Trace On. What part of the problem does the Y2 expression describe? Explain the shape of the graph.

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7) EXIT, deselect Y2 and select Y3. Then DRAW.

What part of the problem does the Y3 expression describe? Trace the graph. Explain the shape of the graph

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8) EXIT, and select all 3 graphs and DRAW. Turn on Trace and use the up-down arrows to toggle between graphs. Is there a relationship between the 3 graphs? Explain.

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9) a) Without using Trace or ISCT, what will be the coordinates of the intersection point of Y1 and Y2? ____________

b) Use Trace and ISCT (on G-Solv) to check your prediction.

10) Use these graphs to find the solution to the problem. Explain your answer.

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11) See if you can find the solution using EQUA. Write your EQUA screen and solution below.

Analysing the Annulus – Solutions and Instructions

Instructions

Q1) The expression is Y = π x 40^2
To enter TABLE mode turn calculator ON, press MENU then 7. delete any pre-existing equations using DEL (F2), then F1
Enter Y = π x 40^2 as per Fig1 **(press SHIFT EXP x 40** 

EXE)

Q2a) Y = π x X^2 **(press SHIFT EXP x** 

EXE) (Fig2)

Q2b) Y = π x 40^2 - π x X^2 **(press SHIFT EXP x 40** 

- SHIFT EXP x 

EXE) (Fig3)

Q3) **Press SET (F5) then enter the table settings as per Fig4**. Using up-down  to move cursor. **Press EXE** after each entry. **Press EXIT.**

Screenshots

Fig1

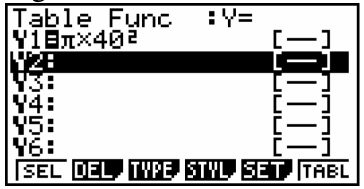


Fig2

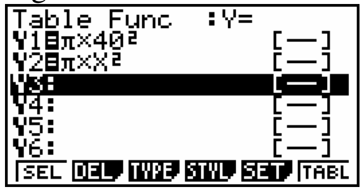


Fig3

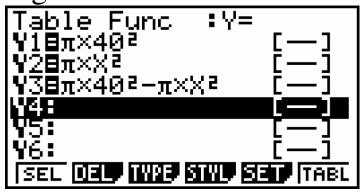
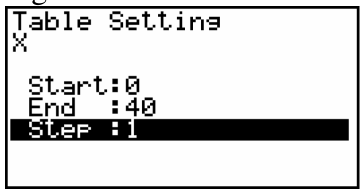


Fig4



Press TABL (F6) (Fig5)

Q3a) Y1 is an expression for the area of the large circle which is a constant (5026.5) (Fig5)

Q3b) Y2 is an expression for the area of the small circle which is zero when the radius (X) is zero and increases to 5026.5 when X is 40

Q3c) Y3 is an expression for the area of the annulus. When the radius of the small circle is zero the annulus has an area equal to the large circle (5026.5). As the radius of the small circle increases the annulus decreases in area becoming zero when the radius of the small circle is 40.

Q3d) The Y2 and Y3 values are linked to the Y1 values in that each Y1 value is the sum of the corresponding Y2 and Y3 values. This is because the large circle area (Y1) is comprised of the small circle area (Y2) and the annulus area (Y3)

Q3e) The approximate solution (the value for X) can be found by looking for when the Y2 and Y3 values are equal. Fig 6 shows that this occurs when X is slightly more than 28
Note: In TABLE mode we can 'drill down' to find more accurate solutions. We know our solution is between 28 and 29.
Press EXIT SET (F5) and then reset the values according to Fig7.
Press EXIT and TABL (F6) and scroll down for the more accurate solution.

Fig8 illustrates that the solution for X is between 28.28 and 28.29. We could 'drill down' further if we wanted to.

Q4) **Go to V-Window (SHIFT F3)** and enter the X and Y max and min values as per Fig9. Ignore scale, dot and other settings.
NOTE: The Y min and max values (-1000 rather than zero and 7000 rather than 5026.5) enable the entire graph to display when being traced.

Fig5

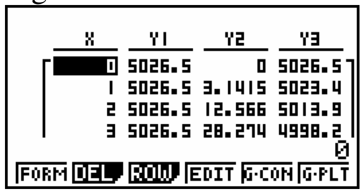


Fig6

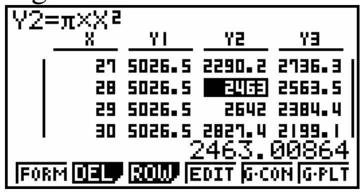


Fig7

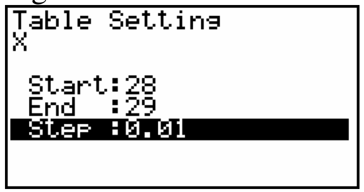


Fig8

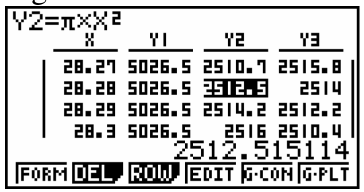
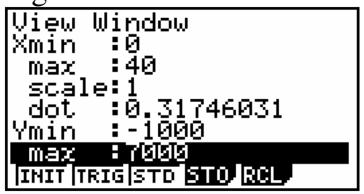
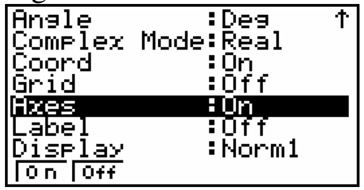


Fig9



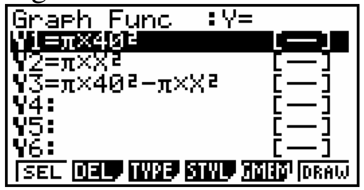
Press EXIT. To turn axes and coordinates On go to **SET UP (SHIFT MENU),** arrow up and turn Axes and Coord On using F1, as in Fig10.

Fig10



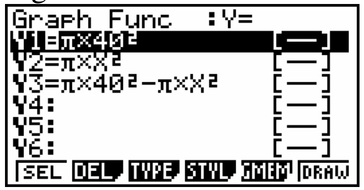
Q5) **Press EXIT then MENU then 5** to go to GRAPH mode.
Note that none of the expressions are selected (Fig11)

Fig11



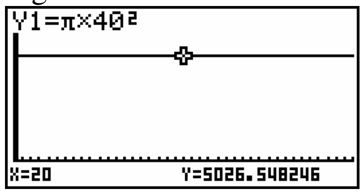
With the cursor over Y1 **press SEL (F1)** (Fig12) <u>LEAVE Y2 AND Y3 UNSELECTED</u>

Fig12



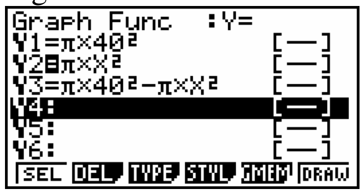
Press DRAW (F6) then turn the trace on with **SHIFT F1** (Fig13)
We can see that this is the graph of the (constant) area of the large circle, which is 5026.5 sq cm. Therefore the graph is a straight line.

Fig13



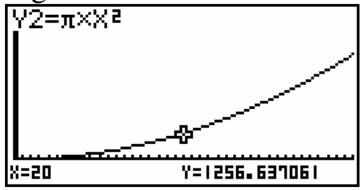
Q6) **Press EXIT, deselect Y1 (press F1)** move the cursor down to Y2 and select with F1 (Fig14)

Fig14



Press DRAW (F6) then Trace (SHIFT F1) (Fig15)

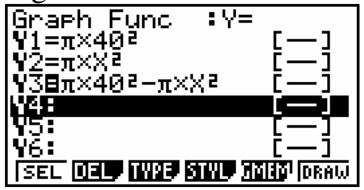
Fig15



The Y2 graph describes the area of the small circle which increases from zero (when the radius is zero) to the area of the large circle (5026.5 when X = 40)

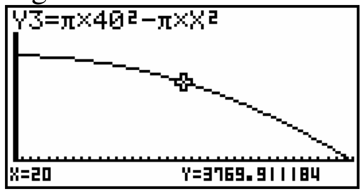
Q7) Press EXIT, deselect Y2 (F1) then select Y3 (F1) (Fig16)

Fig16



Press DRAW (F6) then Trace (SHIFT F1) (Fig17)

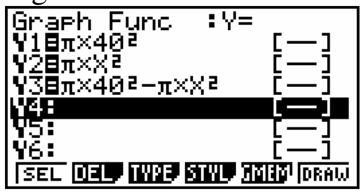
Fig17



Y3 describes the area of the annulus which starts at 5026.5 (the area of the large circle when X = zero) and decreases to zero (when X = 40)

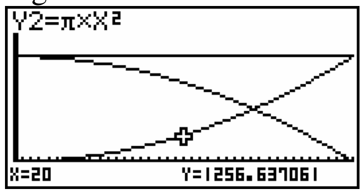
Q8) **Press EXIT and select all 3 graphs using F1** (Fig18)

Fig18



Press DRAW (F6) then Trace (SHIFT F1). Use up-down arrow to toggle between graphs (Fig19)
The relationship between the 3 graphs (the same relationship as was seen from the table) is that each Y1 value is the sum of the corresponding Y2 and Y3 values. This is because the large circle is made up of the small circle plus the annulus.

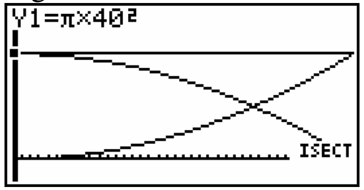
Fig19



Q9a)The coordinates will be (40, 5026.5)

Q9b) In this case the trace will give the exact intersection point (it usually will only give an approximate solution).
However, to use ISCT on G-Solv follow these steps:
Go to G-Solv (SHIFT F5) and press ISCT (F5) (Fig20)

Fig20



The calculator is 'asking' if Y1 is one of the graphs required for the intersection point. (It is)

Press EXE to select Y1 (Fig21). The calculator is now 'asking ' if Y2 is the other graph required for the intersection point.. (It is) **Press EXE to select Y2** (Fig22)
We can see the intersection point is X = 40, Y = 5026.5

Fig21

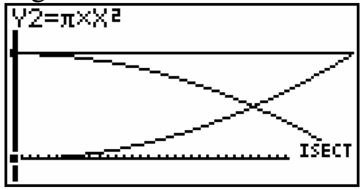
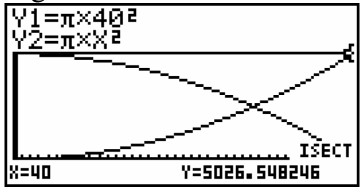


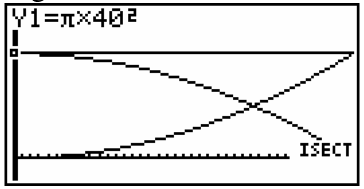
Fig22



Q10) To find the graphical solution we need to find when Y2 (small circle area) is equal to Y3 (annulus area). ie we need to find the intersection of Y2 and Y3 and read off the X value.

Press EXIT then DRAW (F6) then G-Solv (SHIFT F5) then ISCT (F5) (Fig23)

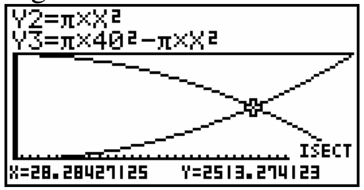
Fig23



We want the intersection of Y2 and Y3. The calculator is 'asking' if Y1 is one of the graphs generating the intersection point (it isn't)

Press down arrow and select graphs Y2 and Y3, **pressing EXE** each time (Fig24)

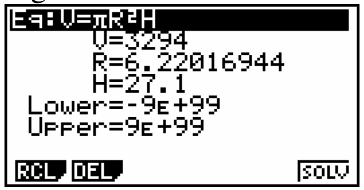
Fig24



The solution to the problem is X = 28.28 (as per the table) The graph shows that when X = 28.28 the area of the annulus is equal to the area of the small circle.

Q11) To enter EQUA **press MENU, scroll to EQUA and EXE. Then press Solver (F3)** (Fig25)
You will probably have a past entry displayed. (Different to Fig25)

Fig25



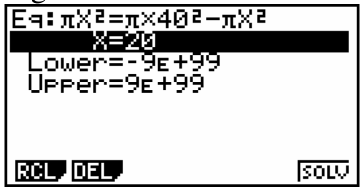
Press DEL (F2) then F1 (Fig26)

Fig26

Enter the equation as per Fig27 (ie small circle = annulus) and **press EXE** (Fig27)
NOTE: The value of X showing is NOT the solution!! Yours will likely be different to Fig27

To solve the equation place the **cursor over the pro-numeral to be solved (X in this case) and press SOLV (F6)**

Fig27



Our familiar solution of 28.28 can be viewed as per Fig 28 OR press EXIT and view the solution in the previous screen.
The Lft and Rgt settings (Fig28) simply indicate the solution has resulted in a balanced equation.

Fig28