

# The Circumference Sum Investigation –

## A note to teachers:

This is a 'quirky algebraic modelling' investigation. That is to say a 'quirky' problem, rather than 'real world' problem, generates the model.

It is important to keep in mind the purposes of running algebraic modelling investigations such as this given the solution has negligible 'real world' importance.

The purposes of this activity include:

- To engage students mathematically
  
- To give students experience of applying algebra to a problem
  
- To efficiently generate a table of values:
  - To enable students to 'see the problem' represented in the numbers
  - To expose students to finding a solution within a table
  
- To efficiently generate graph/s for the problem:
  - To enable students to 'see the problem' represented in the graph/s
  - To find a graphical solution

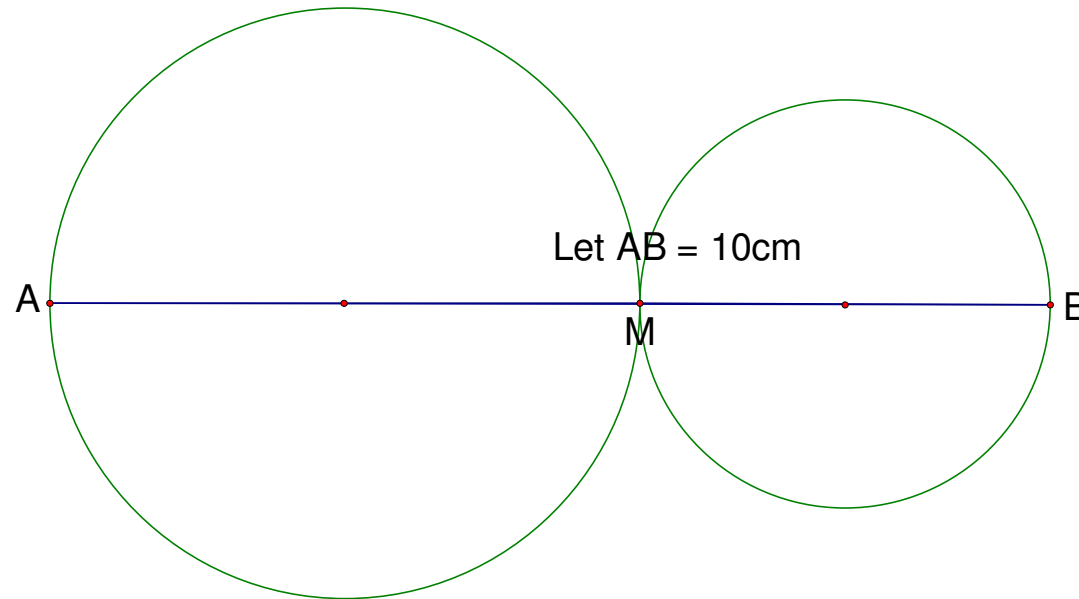
(And possibly the most important purpose of this activity)

- To create numerous opportunities for the teacher to ask 'Working Mathematically' questions during the activity, thereby giving students many opportunities to think mathematically, make mental links and gain conceptual understanding.

Note: A common approach to these 'quirky' algebraic modelling investigations is to first generate a table, check for an approximate solution and then generate the graph/s. However, with 'Circumference Sum' it is recommended to generate the graph first. This is because the result to most people is surprising. Seeing the graphical representation first (prior to the table) capitalizes on this element of surprise!

NOTE: If you are new to TABLE and GRAPH modes you may find it beneficial to first work through the worksheet 'Self-Guided\_9860\_TABLE-GRAPH'.

## Algebraic Modelling The Circumference Sum Investigation



Point M moves along AB simultaneously enlarging one circle and reducing the other.

AB is a fixed line segment of length 10cm.

**The Challenge:** Where does M need to be along AB for the sum of the circumferences to be the greatest?

1) Write down the number of cm from A you think M needs to be in order for the combined circumference to be the greatest. Remember length AB = 10cm. \_\_\_\_\_

2) Set up an equation for the combined circumference. Write the equation here. \_\_\_\_\_

3) Convert the equation to a 'Y =' with the other variable being 'X', making it suitable for GRAPH and TABLE mode.  
Write the equation here. \_\_\_\_\_

4) Set up the axes appropriately using V-Window

5) Turn the axes ON in SETUP

6) Enter the equation into the GRAPH screen

7) Draw the graph

8) Trace the graph. Does the graph make sense? What does the graph tell you about the combined circumference?

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9) Enter TABLE mode and use SET to set up an appropriate table

10) Generate the table. Does the table make sense? Does the table confirm your conclusions from your graph?

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11) Simplify the initial formula algebraically to demonstrate the solution another way. \_\_\_\_\_

# Circumference Sum – Solutions and Instructions

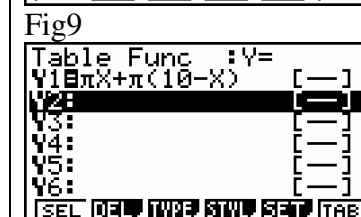
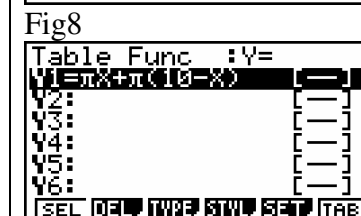
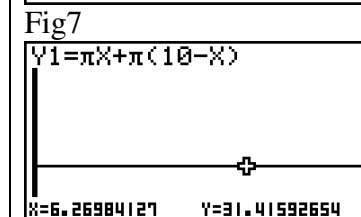
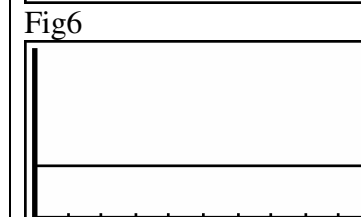
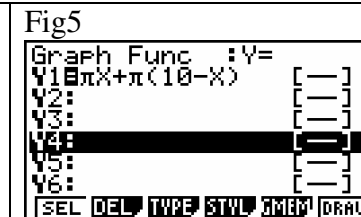
Instructions	Screenshots
<p>Q2) Let one diameter be D. The other diameter therefore becomes (10-D). The combined circumference = <math>\pi D + \pi(10 - D)</math></p> <p>Q3) <math>Y = \pi X + \pi(10 - X)</math></p> <p>Q4) <b>Turn the calculator ON, press MENU and scroll to GRAPH (or press 5) and press EXE</b> (Fig1). NOTE: There may be expressions showing from prior exercises. To set up the axes <b>select V-Window (SHIFT F3)</b> (Fig2) IGNORE CURRENT SETTINGS We need to enter the maximum and minimum values for X and Y. IGNORE THE SCALE AND DOT SETTINGS. <b>Enter Xmin=0, Xmax=10, Ymin=0, Ymax=100</b> (as in Fig3) NOTE: IT DOES NOT MATTER IF YOUR VALUES FOR SCALE AND DOT ARE DIFFERENT TO THOSE SHOWN IN Fig3.</p> <p>Q5) To turn axes ON, press SHIFT MENU, scroll up 3 places, turn axes ON using F1 if necessary. Enter the settings for COORD, GRID and LABEL as per Fig4.  (Setting COORD to On allows the trace to be activated. Setting Label to On simply adds an X and a Y to the screen)</p>	<div style="border: 1px solid black; padding: 5px; margin-bottom: 5px;"> <p>Fig1</p> </div> <div style="border: 1px solid black; padding: 5px; margin-bottom: 5px;"> <p>Fig2</p> </div> <div style="border: 1px solid black; padding: 5px; margin-bottom: 5px;"> <p>Fig3</p> </div> <div style="border: 1px solid black; padding: 5px;"> <p>Fig4</p> </div>

Q6) **Press EXIT**. Delete any equations on the screen by using **F2 and F1** for each. Remembering that 'Y' and '=' already exist in each line of the Graph Function screen, enter  $\pi X + \pi(10 - X)$  from Q2) into the top line (Fig5) NOTE: Make sure the equation is selected. **Scroll down onto a lower line** (Fig5) so that you can see the 'bold box' surrounding the '=' sign. A bold '=' sign means the equation is selected. Make sure Y1 is selected.

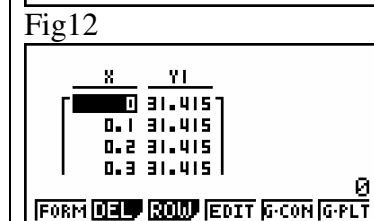
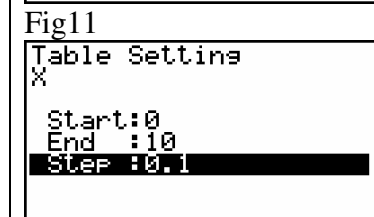
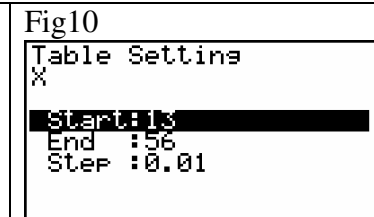
Q7) **Press DRAW** (F6) (Fig6)

Q8) **Press Trace (SHIFT F1) Scroll left and right**. The combined circumference is a constant!! (Fig7)

Q9) **Press MENU, scroll to TABLE (or press 7), and press EXE**. (Fig8) Note that the equation is already copied into TABLE. This is because TABLE and GRAPH modes are always linked on the calculator. Also note that the equation is NOT selected. **Press SEL (F1)** to select the equation (Fig9)



Q10) **Press SET (F5)** (Fig10) NOTE: ignore the current values in the table setting. Yours will be different to Fig10. The Table Setting screen allows you to 'tell the calculator' what values of X will be used from which to generate Y values. We need X values between 0 and 10. The Step setting dictates the difference between each X value. Enter the values as per Fig11. The step of 0.1 will, in this case, give us 100 values. NOTE: The maximum number of values that can be generated in a table is around 250.



**Press EXIT and TABL (F6)** (Fig11) The table confirms the graph - the combined circumference is a constant!

Note that the number should look, to the students, "a bit like pi". In fact it is pi multiplied by 10.

$$\begin{aligned}
 \text{Q11) } Y &= \pi \times X + \pi \times (10-X) \\
 &= \pi X + 10\pi - \pi X \\
 &= 10\pi
 \end{aligned}$$